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Graviton scattering on D6-branes with B-fields

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ABSTRACT: We consider systems of D6-branes in the presence of a non-zero B-field of different ranks. We study the scattering of gravitons in the corresponding supergravity backgrounds. We show that the non-zero B-field does not modify the form of the scattering potential. The graviton scattering equation has two solutions one normalizable and one non normalizable. The normalizable solution does not lead to an absorption, however the non-normalizable one does. We analyse the absorption of gravitons by the branes. We show that in the decoupling limit the graviton flux near the branes is zero, but the absorption is not. This result suggests that even in the presence of a B-field the D6-branes worldvolume theory does not decouple from the bulk gravity. For comparison we analyse the form of the scattering potential and absorption for Dp-branes with $p < 5$ and for NS5-branes.

KEYWORDS: D-branes, AdS-CFT Correspondance.

JHEP06(2000)002

Contents

1. Introduction

The AdS/CFT correspondence (see [1] for a review) relates field theories without gravity to supergravity (string) theories on certain curved backgrounds. The correspondence naturally arises when considering Dp-branes in a limit where the worldvolume field theory decouples from the bulk gravity [2]. As discussed in [3], when turning on a B-field on the Dp-brane worldvolume the low energy effective worldvolume theory is deformed to a non-commutative Super Yang-Mills (NCSYM) theory. With N coinciding Dp-branes in the presence of a non-zero B-field the worldvolume theory is deformed to a U(N) NCSYM [4].

Turning on a B-field on the Dp-brane worldvolume can be viewed via the AdS/CFT correspondence as a perturbation of the worldvolume field theory by a higher dimension operator. The non-commutative effects are relevant in the UV and are negligible in the IR. As in the cases with $B = 0$, there exists a limit where the bulk gravity decouples from the worldvolume non-commutative field theory [5, 4], and a correspondence between string theory on curved backgrounds with B-field and non-commutative field theories is expected.

JHEP06(2000)002

With a vanishing B-field the worldvolume theory of N D6-branes of type-IIA string theory, (in general Dp-branes with $p > 5$), does not decouple from the bulk. There have been indications that in the presence of a non-zero B-field there is a limit where the worldvolume theory of Dp-branes with $p > 5$ may decouple from gravity. In particular, for D6-branes with two non-commutative coordinates there seemed to be for finite N a UV description in terms of eleven-dimensional supergravity on a curved space, while for four or six non-commutative coordinates at finite N a UV description in terms of ten-dimensional supergravity on a curved space [6]. On the other hand, the D6-branes system has a negative specific heat, which is usually taken as a sign of non decoupling of gravity. Thus, the issue whether a non-commutative seven-dimensional field theory on the worldvolume of D6-branes exists without gravity at all energy scales is still unsettled.

In this paper we will address this question by considering systems of D6-branes in the presence of a non-zero B-field of different ranks and analyse the scattering of gravitons in the corresponding supergravity backgrounds. Generically, the sign of decoupling from the bulk is the vanishing of the absorption cross section in the decoupling limit. We will analyse the absorption of gravitons by the branes and show that in the decoupling limit the graviton flux near the branes is zero but the absorption is not zero. For comparison we analyse the form of the scattering potential and absorption for Dp-branes with $p < 5$ and for NS5-branes, where worldvolume theories decoupled from the bulk exist.

The paper is organized as follows. In the next section we review the decoupling limit with and without a B-field. The analysis suggests a possibility of decoupling of D6-branes with a constant NS B-field on their worldvolume from the bulk gravity. In the following sections we will study this issue. In section 3 we will consider systems of D6-branes with and without a B-field. We will analyse the scattering of gravitons in the corresponding supergravity backgrounds. We will obtain the graviton scattering equation and show that the non-zero B-field does not modify the form of the scattering potential. We will then solve the graviton scattering equation. It has two solutions one normalizable and one non normalizable. The normalizable solution does not lead to an absorption, however the non-normalizable one does. We show that in the decoupling limit the graviton flux near the branes is zero. Nevertheless, we argue that the absorption is not zero. The result indicates the non decoupling of the bulk gravity from the brane modes.

The D6-branes supergravity background has a time-like singularity at the origin. We will discuss this singularity and possible resolutions of it. In section 4 we analyse the form of the scattering potential and absorption for Dp-branes with $p < 5$ and for NS5-branes. Our interest in these cases is for comparison with the D6-branes cases. The Dp-branes supergravity backgrounds with $p > 3$ have a curvature singularity at the origin, and when $p < 3$ a dilaton blow-up. In these cases the non-normalizable solution of the graviton scattering equation leads to an absorption.

However, we will see clear differences in the form of the scattering potential and the graviton absorption and their behaviour in the decoupling limit compared to the D6-branes case.

Other papers discussing various aspects of the supergravity backgrounds with B-field in the context of dual descriptions of non-commutative gauge theories are [7]–[14]. Some absorption cross sections in the background of D3-branes with B-field have been computed in [15, 16].

2. The decoupling limit

In the following we will review the decoupling limits for Dp-branes with and without a background of NS B-field [2, 1]. Consider a system of Dp-branes extended along a $(p+1)$-dimensional plane in $(9+1)$-dimensional space-time. At low energies $E < 1/l_s$ only the massless string states are excited. The bulk modes are the massless closed string states that include the graviton, and the branes modes are the massless open string states that include the gauge fields. Consider first the case with a vanishing B-field. The leading terms in the interaction action between the brane modes and the bulk modes are obtained by covariantizing the brane action. The quadratic term in the gauge field strength is

$$S \sim \frac{1}{g_{\rm YM}^2} \int \sqrt{g}\, d^{p+1}x\, g_{\alpha\beta}\, g_{\gamma\delta}\, F^{\alpha\gamma}\, F^{\beta\delta} \,, \tag{2.1}$$

where $g_{\rm YM}^2 = g_s\, l_s^{p-3}$ is the Yang-Mills gauge coupling. To be precise we have to add the dilaton field in (2.1), however this does not affect the following discussion. We expand $g_{\alpha\beta} = \eta_{\alpha\beta} + \kappa_{10}\, h_{\alpha\beta}$ where $\kappa_{10} = l_{10}^4 = g_s\, l_s^4$, and l_{10} is the ten-dimensional Planck scale. In this notation h is canonically normalized.

The action (2.1) is now $S = S_{\rm gauge} + S_{\rm int}$ where

$$\begin{aligned} S_{\rm gauge} &\sim \frac{1}{g_s\, l_s^{p-3}} \int d^{p+1}x\, F_{\mu\nu}\, F^{\mu\nu} \,, \\ S_{\rm int} &\sim \frac{1}{g_s\, l_s^{p-3}} \int d^{p+1}x \Big(\kappa_{10}\, \eta_{\alpha\beta}\, h_{\gamma\delta}\, F^{\alpha\gamma}\, F^{\beta\delta} + O\left(h^2\right) \Big) \,. \end{aligned} \tag{2.2}$$

$S_{\rm gauge}$ is the action of the worldvolume gauge theory and $S_{\rm int}$ is the interaction action with the bulk gravity.

The decoupling limit of Dp-branes from the bulk is the low energy limit. In the decoupling limit we send $l_s \to 0$ and keep the Yang-Mills coupling $g_{\rm YM}^2 = g_s\, l_s^{p-3}$ fixed. When $p < 6$, the ten-dimensional Planck length $l_{10} = g_s l_s^{1/4}$ vanishes implying that the interaction action (2.2) vanishes. We should worry about the value of the eleven-dimensional Planck length $l_{11} = g_s^{1/3} l_s$ as well since at some energy scale we may need to pass to an eleven-dimensional description of the system where l_{11} appears in the interaction action and not l_{10}. This happens for D2-branes at low energy and for D4-branes at high energy [17]. When $p < 6$, the eleven-dimensional Planck length vanishes too in the above limit implying that the interaction action vanishes in the eleven-dimensional background. In these cases the field theories on the branes decouple from the bulk.

When $p = 6$ we keep the Yang-Mills coupling $g_{\rm YM}^2 = g_s\, l_s^3$ fixed, which means that we keep the eleven-dimensional Planck length l_{11} fixed. The phase diagram of the D6-branes system shows that at high energy we have to use an eleven-dimensional description [17]. In view of the above discussion, the fact that l_{11} is fixed means that interaction action does not vanish and the bulk gravity does not decouple from the field theory on the branes. For NS-branes the decoupling limit of from the bulk is the limit of weak coupling of the bulk physics $g_s \to 0$, while keeping l_s fixed [18].

Consider now the case where we also have a background of constant NS B-field on the D-branes worldvolume. In this case the decoupling limit is different. In this set up, the end points of the open strings attached to the branes, x_i, are non commuting. Consider this system in the extreme condition where $B_{i,i+1} \to \infty$ as $l_s \to 0$ such that $b_i \equiv l_s^2\, B_{i,i+1}$ is fixed. Upon rescaling the coordinates $x_i \to \frac{b_i}{l_s^2}\, x_i$ and keeping the new coordinates fixed in the limit one gets

$$[x_i, x_{i+1}] = i\, b_i\, . \tag{2.3}$$

In the presence of the B-field, the massless states excitations of the open strings attached to the Dp-branes give rise to a non-commutative worldvolume field theories, with b_i (2.3) being the deformation parameters.

Consider a B-field of rank $2m$. Now in the decoupling limit we keep $g_{\rm YM}^2 \sim g_s\, l_s^{p-3-2m}$ fixed. Consider the D6-branes in the presence of a B-field. In this case the rank of the B-field can be up to six, $m = 1, 2, 3$. When $m = 1$ we need an eleven-dimensional description in the UV [6]. Since in the decoupling limit we keep $g_s\, l_s =$ fixed as $l_s \to 0$, both the ten-dimensional Planck length l_{10} and the eleven-dimensional Planck length l_{11} vanish and it seems that gravity decouples. For $m = 2, 3$ the effective string coupling is small at all energy scales [6] and there are situations with no need for an eleven-dimensional description at high energy. Again, both the ten-dimensional Planck length l_{10} and the eleven-dimensional Planck length l_{11} vanish and it seems that gravity decouples.

We note however that the above argument can fail in the following way. The theory on the branes is a non-commutative field theory which can be recast as a commutative field theory with infinite number of terms in the gauge field strength and its derivatives [4]. With the inclusion of these terms the theory is non local. We have been discussing the coupling to gravity as for a local theory, and analysis of the coupling to gravity term by term suggests that the coupling is l_{10} (or l_{11}). This maybe misleading, and upon adding all the infinite number of terms it is possible that the interaction of the complete non-local theory and gravity is not as simple.

JHEP06(2000)002

As a comparison we can consider the issue of renormalizability of the non-local non-commutative field theory which seems surprising when viewed as a commutative field theory with some higher dimension operators.

3. Graviton scattering on D6-branes

In this section we will study the scattering of gravitons on D6-branes with and without a B-field. We will first compute the scattering potential and then analyse the scattering in that potential. We will start with the analysis of D6-branes with $B = 0$ and continue with the cases of non-vanishing B-field with different ranks.

3.1 D6-branes with $B = 0$

We denote the ten-dimensional coordinates by $x_0, x_1, \ldots, x_9$. Consider N parallel D6-branes stretched in $x_0, \ldots, x_6$. The supergravity solution in the string frame takes the form [19]

$$ds^2 = \sqrt{f(r)} \left(\frac{-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dx_6^2 + dx_5^2}{f(r)} + \right.$$
$$\left. + \; dr^2 + r^2 \left(d\vartheta^2 + \sin^2 \vartheta \, d\phi^2 \right) \right),$$
$$e^{2\Phi} = f(r)^{-3/2}, \qquad A_\phi = -R \cos \vartheta, \qquad f(r) = 1 + \frac{R}{r}, \tag{3.1}$$

where $R \sim g_s \, l_s \, N$. Φ is the dilaton and A_ϕ is the RR 1-form dual to the RR 7-form that couples to the D6-brane worldvolume.

We will perturb the metric of the background (3.1) by

$$g_{ab} = \bar{g}_{ab} + h_{ab}, \quad a, b = 0, \ldots, 9, \tag{3.2}$$

where by $\bar{g}_{ab}$ we denote the background metric (3.1) and h_{ab} is the perturbation. The linearization of the type-IIA field equations is done in appendix A. There are several polarizations ε_{ab} of the graviton that have to be considered. Also, we have to handle the ambiguities in the perturbation (3.2) that arise due to the diffeomorphism invariance by an appropriate gauge fixing. We consider s-wave gravitons with momenta along the brane

$$h_{ab} = \varepsilon_{ab} \, h(r) \, e^{i \, k_\mu x^\mu}, \quad \mu = 0, \ldots, 6. \tag{3.3}$$

The higher partial waves will simply encounter an additional centrifugal potential. We choose the gauge

$$h_{a\mu} \, k^\mu = 0, \tag{3.4}$$

that keeps transversal gravitons. We will choose the gravitons with polarization along the brane, i.e. $\varepsilon_{ab} = 0, a, b = 7, 8, 9$. The other polarizations correspond to

JHEP06(2000)002

vectors and scalars from the worldvolume theory point of view. Let $k_\mu = w\,\delta_{0,\mu}$. We will consider other possibilities later. This implies via (3.4) that $\varepsilon_{a0} = 0$. In addition there is a tracelessness condition on the polarization tensor $\eta^{\mu\nu}\varepsilon_{\mu\nu} = 0$ implied by the linearized field equations. All together we see that there are 20 possible choices of polarizations of $\varepsilon_{\mu\nu}$. They can be realized by 15 off-diagonal configurations such as

$$\varepsilon_{12} = \varepsilon_{21} = 1\,, \qquad \text{else } \varepsilon_{\mu\nu} = 0\,, \tag{3.5}$$

plus 5 diagonal configurations such as

$$\varepsilon_{11} = -\varepsilon_{22} = 1\,, \qquad \text{else } \varepsilon_{\mu\nu} = 0\,. \tag{3.6}$$

Both types of polarizations yield the same equation for $h(r)$, which using the linearized equations of appendix A and the background (3.1) reads

$$\Big(\partial_r^2 + a(r)\,\partial_r + b(r)\Big)h(r) = 0\,, \tag{3.7}$$

with the functions $a(r)$ and $b(r)$ given by

$$a(r) = \frac{2\,r + R}{r(r+R)}\,, \qquad b(r) = \omega^2\left(1 + \frac{R}{r}\right) - \frac{R^2}{4\,r^2(r+R)^2}\,. \tag{3.8}$$

We can write $h(r) = g(r)\,c\,(r)$ with the function $c\,(r)$ given by

$$c\,(r) = \frac{1}{\sqrt{r(r+R)}}\,. \tag{3.9}$$

Now eq. (3.7) can be recast as a schrödinger-like equation for the function $g(r)$ and takes the form

$$\big(\partial_r^2 - V(r)\big)\,g(r) = 0\,, \tag{3.10}$$

with the potential $V(r)$ given by

$$V(r) = -\omega^2\left(1 + \frac{R}{r}\right). \tag{3.11}$$

The potential (3.11) is plotted in figure 1. It is an attractive Coulomb-like potential. In order to get the scattering potential for the higher partial waves of the graviton we simply have to add the centrifugal piece, i.e.

$$V(r) = -\omega^2\left(1 + \frac{R}{r}\right) + \frac{l(l+1)}{r^2}\,. \tag{3.12}$$

We will analyse the scattering in this potential later.

The analysis is rather general and can be easily modified for other perturbations of the D6-branes background. For instance, if instead of a graviton we would have considered a minimally coupled scalar $\varphi(r)$, and decomposed it as $\varphi(r) = g(r)\,c\,(r)$

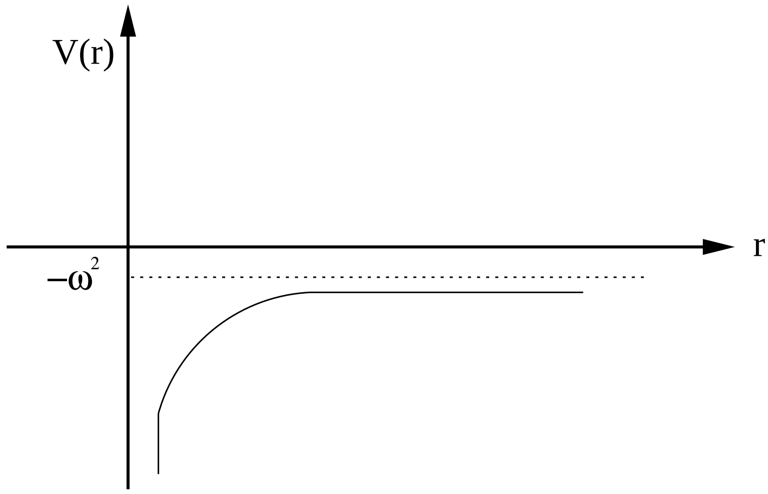

Figure 1: The scattering potential $V(r)$ for gravitons polarized along the D6-brane.

with the function $c(r) = 1/r$ then the equation for the function $g(r)$ would takes form (3.10) with the potential (3.11) for the s-wave mode and (3.12) for the higher partial waves. In general, the transversal graviton modes $h(r)$ are related to the minimally coupled scalar $\varphi(r)$ like

$$h(r) = \frac{1}{\sqrt{f(r)}}\,\varphi(r)\,, \tag{3.13}$$

with $f(r)$ being the harmonic function appearing in the metric, here given in (3.1).

3.2 D6-branes with rank two ($m = 1$) B-field

Consider N parallel D6-branes stretched in $x_0, \ldots, x_6$, with a B-field of rank two. We choose the B-field such that $B_{56} \neq 0$. The supergravity background can be generated by starting with delocalized D5-branes oriented at an angle in the (x_5, x_6)-plane and applying T-duality map on the x_6 coordinate. The background takes the form [20]

$$\begin{aligned}
ds^2 &= \sqrt{f(r)}\left(\frac{-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2}{f(r)} + \frac{dx_6^2 + dx_5^2}{\sin^2\alpha_1 + f(r)\cos^2\alpha_1} + \right.\\
&\qquad\left. +\; dr^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\phi^2\right)\right),\\
e^{2\Phi} &= \frac{1}{\sqrt{f(r)}\left(\sin^2\alpha_1 + f(r)\cos^2\alpha_1\right)}\,, \qquad B_{65} = \frac{(f(r)-1)\,\sin\alpha_1\cos\alpha_1}{\sin^2\alpha_1 + f(r)\cos^2\alpha_1}\,, \\
A_\phi &= -R\cos\vartheta\cos\alpha_1\,, \qquad A_{56\phi} = -\frac{R\,\cos\vartheta\sin\alpha_1}{\sin^2\alpha_1 + f(r)\cos^2\alpha_1}\,, \qquad f(r) = 1 + \frac{R}{r}\,,
\end{aligned} \tag{3.14}$$

where $R \sim g_s\, l_s N \cos^{-1}\alpha_1$. In the limit $\alpha_1 \to 0$ the B-field vanishes and we recover the background (3.1).

There are now two cases to consider. Consider first a graviton polarized along the D6-brane but orthogonal to the B-field. The equation for the perturbation $h(r)$ takes the form (3.7) with $a(r), b(r)$ given by (3.8). Again, writing $h(r) = g(r)\, c\,(r)$ with $c\,(r)$ given by (3.9), the equation for $g(r)$ takes the form (3.10) with the potential (3.11).

Consider next a graviton polarized along the D6-brane parallel to the B-field. The equation for $h(r)$ takes the form (3.7) with the functions $a(r)$ and $b(r)$ given by

$$
\begin{aligned}
a(r) &= \frac{4\,r^2 + 6\,rR + R^2 + R^2\cos 2\alpha_1}{r(r+R)(2r+R+R\cos 2\alpha_1)}\,, \\
b(r) &= \omega^2\left(1+\frac{R}{r}\right) - \frac{R^2(-2\,r+R+(4\,r+R)\cos\,2\alpha_1)}{4\,r^2(r+R)^2\,(2\,r+R+R\cos 2\,\alpha_1)}\,.
\end{aligned} \tag{3.15}
$$

We can write $h(r) = g(r)\, c\,(r)$ where now $c\,(r)$ is given by

$$
c\,(r) = \frac{\sqrt{r+R}}{\sqrt{r}\,(2r+R+R\cos 2\alpha_1)}\,. \tag{3.16}
$$

Now eq. (3.7) can be recast as an equation for the function $g(r)$ and takes the form (3.10) with the potential (3.11).

3.3 D6-branes with rank four ($m = 2$) B-field

Consider N parallel D6-branes stretched in $x_0, \ldots, x_6$, with a B-field of rank four. We choose the B-field such that B_{34} and B_{56} are non zero. The supergravity background can be generated by starting with delocalized D4-branes oriented at angles in the (x_4, x_5, x_6)-plane and applying twice the T-duality map, on the x_5 and x_6 coordinates. The background takes the form

$$
\begin{aligned}
ds^2 &= \sqrt{f(r)}\left(\frac{-dx_0^2+dx_1^2+dx_2^2}{f(r)} + \frac{dx_3^2+dx_4^2}{\sin^2\alpha_1 + f(r)\cos^2\alpha_1} + \right. \\
&\qquad \left. + \frac{dx_5^2+dx_6^2}{\sin^2\alpha_2+f(r)\cos^2\alpha_2} + dr^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\phi^2\right)\right), \\
e^{2\Phi} &= \frac{\sqrt{f(r)}}{\left(\sin^2\alpha_1+f(r)\cos^2\alpha_1\right)\left(\sin^2\alpha_2+f(r)\cos^2\alpha_2\right)}\,, \\
B_{43} &= \frac{(f(r)-1)\sin\alpha_1\cos\alpha_1}{\sin^2\alpha_1+f(r)\cos^2\alpha_1}\,, \qquad B_{65} = \frac{(f(r)-1)\sin\alpha_2\cos\alpha_2}{\sin^2\alpha_2+f(r)\cos^2\alpha_2}\,, \\
A_\phi &= -R\cos\vartheta\cos\alpha_1\cos\alpha_2\,, \qquad A_{012} = -\frac{\sin\alpha_1\sin\alpha_2}{f(r)}\,, \\
A_{56\phi} &= -\frac{R\cos\vartheta\cos\alpha_1\sin\alpha_2}{\sin^2\alpha_2+f(r)\cos^2\alpha_2}\,, \qquad A_{34\phi} = -\frac{R\cos\vartheta\cos\alpha_2\sin\alpha_1}{\sin^2\alpha_1+f(r)\cos^2\alpha_1}\,, \\
f(r) &= 1+\frac{R}{r}\,,
\end{aligned} \tag{3.17}
$$

where $R \sim g_s\, l_s\, N\cos^{-1}\alpha_1\cos^{-1}\alpha_2$. In the limit $\alpha_1 \to 0$ the B_{34} field vanishes and we recover the background (3.14) with rank two B-field.

Consider a graviton polarized along the D6-brane parallel to the B_{56} field. The equation for $h(r)$ takes the form (3.7) with the functions $a(r)$ and $b(r)$ given by

$$
\begin{aligned}
a(r) &= \frac{4\,r^2 + 6\,rR + R^2 + R^2\cos 2\,\alpha_2}{r(r+R)(2\,r + R + R\cos 2\,\alpha_2)}\,, \\
b(r) &= \omega^2\left(1 + \frac{R}{r}\right) - \frac{R^2(-2\,r + R + (4\,r + R)\cos 2\,\alpha_2)}{4\,r^2(r+R)^2\,(2\,r + R + R\cos 2\alpha_2)}\,.
\end{aligned} \tag{3.18}
$$

As before, can write $h(r) = g(r)\,c\,(r)$ where now $c\,(r)$ is given by

$$
c\,(r) = \frac{\sqrt{r+R}}{\sqrt{r}\,(2\,r + R + R\cos 2\alpha_2)} \tag{3.19}
$$

and the equation for the function $g(r)$ takes the form (3.10) with the potential (3.11).

3.4 D6-branes with rank six ($m = 3$) B-field

The highest rank for the B-field on the worldvolume of D6-branes is six. Consider now this case. All the previous cases are special limits of the current one. Let again the N parallel D6-branes be stretched in $x_0, \ldots, x_6$, with a B-field of configuration of rank six, which we choose such that B_{12}, B_{34} and B_{56} are non zero. The supergravity background can be generated by staring with delocalized D3-branes oriented at angles in the (x_3, x_4, x_5, x_6)-plane and applying three times the T-duality map, on the x_4, x_5 and x_6 coordinates. The background takes the form

$$
\begin{aligned}
ds^2 &= \sqrt{f(r)}\left(\frac{-dx_0^2}{f(r)} + \frac{dx_1^2 + dx_2^2}{\sin^2\alpha_1 + f(r)\cos^2\alpha_1} + \frac{dx_3^2 + dx_4^2}{\sin^2\alpha_2 + f(r)\cos^2\alpha_2} + \right. \\
&\qquad \left. + \frac{dx_5^2 + dx_6^2}{\sin^2\alpha_3 + f(r)\cos^2\alpha_3} + dr^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\phi^2\right)\right), \\
e^{2\Phi} &= \frac{f(r)^{3/2}}{\prod_{i=1}^{3}\left(\sin^2\alpha_i + f(r)\cos^2\alpha_i\right)}\,, \qquad B_{21} = \frac{(f(r)-1)\sin\alpha_1\cos\alpha_1}{\sin^2\alpha_1 + f(r)\cos^2\alpha_1}\,, \\
B_{43} &= \frac{(f(r)-1)\sin\alpha_2\cos\alpha_2}{\sin^2\alpha_2 + f(r)\cos^2\alpha_2}\,, \qquad B_{65} = \frac{(f(r)-1)\sin\alpha_3\cos\alpha_3}{\sin^2\alpha_3 + f(r)\cos^2\alpha_3}\,, \\
A_0 &= \left(\frac{1}{f(r)} - 1\right)\sin\alpha_1\sin\alpha_2\sin\alpha_3\,, \qquad A_\varphi = -R\cos\vartheta\cos\alpha_1\cos\alpha_2\cos\alpha_3\,, \\
A_{012} &= \frac{(f(r)-1)\sin\alpha_2\sin\alpha_3\cos\alpha_1}{\sin^2\alpha_1 + f(r)\cos^2\alpha_1}\,, \qquad A_{12\,\varphi} = -\frac{R\cos\vartheta\cos\alpha_2\cos\alpha_3\sin\alpha_1}{\sin^2\alpha_1 + f(r)\cos^2\alpha_1}\,, \\
A_{034} &= \frac{(f(r)-1)\sin\alpha_1\sin\alpha_3\cos\alpha_2}{\sin^2\alpha_2 + f(r)\cos^2\alpha_2}\,, \qquad A_{34\,\varphi} = -\frac{R\cos\vartheta\cos\alpha_1\cos\alpha_3\sin\alpha_2}{\sin^2\alpha_2 + f(r)\cos^2\alpha_2}\,, \\
A_{056} &= \frac{(f(r)-1)\sin\alpha_1\sin\alpha_2\cos\alpha_3}{\sin^2\alpha_3 + f(r)\cos^2\alpha_3}\,, \qquad A_{56\,\varphi} = -\frac{R\cos\vartheta\cos\alpha_1\cos\alpha_2\sin\alpha_3}{\sin^2\alpha_3 + f(r)\cos^2\alpha_3}\,, \\
f(r) &= 1 + \frac{R}{r}\,,
\end{aligned} \tag{3.20}
$$

where $R \sim g_s\, l_s\, N \prod_{i=1}^{3} \cos^{-1}\alpha_i$. In the limit $\alpha_1 \to 0$ the B_{12} field vanishes and we recover the background (3.17) with rank two B-field.

As a more general example, consider now a graviton polarized parallel to the B_{56} field with momenta k_1 and k_2 along B_{12} and B_{34}, respectively. The equation for $h(r)$ takes the form (3.7) with the functions $a(r)$ and $b(r)$ given by

$$
\begin{aligned}
a(r) &= \frac{4\,r^2 + 6\,rR + R^2 + R^2\cos 2\,\alpha_3}{r(r+R)(2\,r + R + R\cos 2\,\alpha_3)}\,, \\
b(r) &= \left(\omega^2\left(1+\frac{R}{r}\right) - k_1^2\left(1+\frac{R\cos^2\alpha_1}{r}\right) - k_2^2\left(1+\frac{R\cos^2\alpha_2}{r}\right)\right) - \\
&\quad - \frac{R^2(-2r + R + (4\,r + R)\cos 2\alpha_3)}{4\,r^2(r+R)^2(2\,r + R + R\cos 2\,\alpha_3)}\,.
\end{aligned}
\tag{3.21}
$$

As before, can write $h(r) = g(r)\,c\,(r)$ where now $c\,(r)$ is given by

$$
c\,(r) = \frac{\sqrt{r+R}}{\sqrt{r}\,(2\,r + R + R\cos 2\,\alpha_3)}\,. \tag{3.22}
$$

The equation for the function $g(r)$ takes the form (3.10) with the potential

$$
V(r) = -\left(1+\frac{R}{r}\right)\omega^2 + k_1^2\left(1+\frac{R\cos^2\alpha_1}{r}\right) + k_2^2\left(1+\frac{R\cos^2\alpha_2}{r}\right), \tag{3.23}
$$

with $\omega^2 - k_1^2 - k_2^2 \geq 0$. We see that the momenta k_1, k_2 do not change the structure of the potential. In particular, we can recast (3.23) in the form (3.11) by redefining ω and R.

3.5 Graviton scattering

Let us summarize the information gained in the above analysis. We considered a graviton polarized along the D6-brane worldvolume with $k_\mu = w\,\delta_{0,\mu}$. We saw that if we decompose the graviton $h(r) = g(r)\,c\,(r)$ then with an appropriate choice of $c\,(r)$ the equation for $g(r)$ takes the form (3.10) with the potential (3.11) for the s-wave and (3.12) for the higher partial waves. The only difference between having a non-vanishing B-field or not is reflected in the form of the function $c\,(r)$. The differential equation (3.10) with the potential (3.12) can be solved exactly. It has two solutions

$$
\begin{aligned}
g_1(r) &= (\omega\,r)^{l+1}\,e^{i\omega r}\,{}_1F_1\left(l+1-\frac{i\,\omega R}{2}, 2+2\,l; -2\,i\,\omega\,r\right), \\
g_2(r) &= (\omega\,r)^{l+1}\,e^{i\omega r}\,U\left(l+1-\frac{i\,\omega R}{2}, 2+2\,l; -2\,i\,\omega\,r\right),
\end{aligned}
\tag{3.24}
$$

where ${}_1F_1$ and U are the Kummer confluent hypergeometric functions. Consider the s-wave. The asymptotic behaviour of the functions $g_1(r)$ and $g_2(r)$ as $r \to \infty$ and $r \to 0$ is

$$
\begin{aligned}
g_1(r \longrightarrow \infty) &\sim e^{-i\pi(1-i\omega R/2)}(-2\,i\,\omega\,r)^{i\omega R/2}\,\frac{e^{i\omega r}}{\Gamma\,[1+i\,\omega R/2]} + \\
&\qquad + (-2\,i\,\omega\,r)^{-i\omega R/2}\,\frac{e^{-i\omega r}}{\Gamma\,[1-i\,\omega\,R/2]}\,, \\
g_2(r \longrightarrow \infty) &\sim (-2\,i\,\omega\,r)^{i\omega R/2}\,e^{i\omega r}
\end{aligned}
\tag{3.25}
$$

and

$$g_1(r \longrightarrow 0) \sim \omega\, r\, e^{i\omega r}\,, \qquad g_2(r \longrightarrow 0) \sim \frac{2\, i\, e^{i\omega r}}{\Gamma\left[1 - i\,\omega R/2\right]}\,. \tag{3.26}$$

Both $g_1(r)$ and $g_2(r)$ are regular solutions. However, we should recall that the graviton function is $h(r) = g(r)\, c\,(r)$. As $r \to 0$ the function $c\,(r) \sim 1/\sqrt{r}$ independently of the rank of the B-field. Thus, while $h_1(r) = g_1(r)\, c\,(r)$ is regular at the origin, $h_2(r) = g_2(r)\, c\,(r)$ diverges there. Recall that $h_{ab} = \varepsilon_{ab}\, h(r) e^{i\omega t}$ with the polarizations ε_{ab} analysed previously. Then, with respect to the action measure

$$||h||^2 \sim \int dr \sqrt{g}\, e^{-2\Phi}\, g^{ab}\, g^{cd}\, g^{rr} \Big(\partial_r\, h^*_{ac}(r) \partial_r\, h_{bd}(r) + \omega^2\, g^{tt}\, h^*_{ac}\,(r)\, h_{bd}\,(r) \Big)\,, \tag{3.27}$$

$h_1(r)$ is normalizable while $h_2(r)$ is non normalizable,

Denote by $e^{i\omega r}$ an incoming wave and by $e^{-i\omega r}$ an outgoing wave. From the expansion near infinity (3.25) we see that $h_1(r)$ consists of both incoming and outgoing waves while $h_2(r)$ is an incoming wave. From the expansion near zero (3.26) we see that both $h_1(r)$ and $h_2(r)$ are of the form of an incoming wave.

Consider first the normalizable solution $h_1(r)$. We can read the scattering cross section from (3.25) or alternatively we can compute the flux by

$$\mathcal{F} \sim \frac{1}{2\, i} \int \sqrt{g}\, e^{-2\Phi}\, g^{ab}\, g^{cd}\, g^{rr} \Big(h^*_{ac}(r)\, \partial_r\, h_{bd}(r) - \partial_r\, h^*_{ac}(h)\, h_{bd}(r) \Big)\,. \tag{3.28}$$

We denote the incoming and outgoing fluxes at infinity by $\mathcal{F}^{\rm in}_\infty$ and $\mathcal{F}^{\rm out}_\infty$, respectively, and by $\mathcal{F}^{\rm in}_0$ the incoming flux at zero. We see that if we only consider the normalizable solution $h_1(r)$ we have

$$\frac{\mathcal{F}^{\rm out}_\infty}{\mathcal{F}^{\rm in}_\infty} = 1\,, \qquad \mathcal{F}^{\rm in}_0 = 0\,. \tag{3.29}$$

That means that there is no absorption of gravitons by the D6-branes and all the gravitons are reflected back. This is the familiar Rutherford scattering in a Coulomb potential. Indeed in the analysis of scattering in a Coulomb potential, only the normalizable solution is relevant. The argument invoked in discarding the non-normalizable solution is that it requires a δ-function source at the origin, and such source does not exist. Thus, there is no absorption by the point-like charged source at the origin and all the waves are scattered back.

This is not the physics of our system of D6-branes. At least before taking the decoupling limit we expect gravitons to be absorbed by the branes and excite the brane modes such as the gauge fields. This implies that we have to consider the non-normalizable solution as well. In the next section we will see that this is also required for other Dp-brane when $p \neq 3$. The reason why we do have to consider in all these cases the non-normalizable solution is the fact that we have a singularity at $r = 0$, due to the the Dp-branes source. Let us discuss this issue in some more detail.

The singularity of the D6-branes supergravity background at $r = 0$ is time-like. Having such a singularity of the classical geometry which we can reach at a finite proper time, there is the natural question whether it is resolved quantum mechanically. One criterion [21] is the existence of a self-adjoint laplacian. This can still be the case even if the metric is geodesically incomplete. Indeed the requirement is the existence of a non-normalizable solution of the wave equation. This criterion is satisfied by our geometry. Physically we expect that the singularity is resolved by the D6-branes source. The details of the resolution may differ in the presence or absence of a B-field, since it is related to the branes degrees of freedom which are missed by the classical supergravity background. The field theory on the branes differ with a non-vanishing B-field compared to $B = 0$. The non-normalizable solution is the probe we have on the physics at $r = 0$.

Consider now the non-normalizable solution $h_2(r)$. We denote the incoming flux at infinity by $\mathcal{G}^{\rm in}_\infty$ and by $\mathcal{G}^{\rm in}_0$ the incoming flux at zero. Again, we can compute the ratio of fluxes by using the wave function $h_2(r)$ or by computing the fluxes using (3.28). In both cases we get

$$\frac{\mathcal{G}^{\rm in}_0}{\mathcal{G}^{\rm in}_\infty} = e^{-\pi\omega R/2}\, \frac{\sinh\left(\pi\,\omega\, R/2\right)}{\pi\,\omega\, R/2}\,. \tag{3.30}$$

In the decoupling limit $\omega\, l_s \to 0$, $\alpha_i \to \pi/2$ and we keep $b_i = l_s^2 \cos^{-1}\alpha_i$ and $g_s\, l_s^{p-3-2m}$ fixed. Since $R \sim g_s\, l_s N \prod_{i=1}^{3} \cos^{-1}(\alpha_i)$, we see that $\omega R \to \infty$. Equation (3.30) implies then that $\mathcal{G}^{\rm in}_0/\mathcal{G}^{\rm in}_\infty \to 0$. This seems to be a signal of decoupling. However, we run now into some puzzles. The first is that the above analysis does not distinguish the cases with or without a B-field, while we know that in the absence of a B-field the D6-branes do not decouple from the bulk. This puzzle could be resolved by noting that one of our assumptions was that we can use the ten-dimensional supergravity background at all energy scales. This is not correct for D-branes in the absence of a B-field, since at some energy scale we have to use the eleven-dimensional description. Indeed, if we recall that without a B-field we keep in the decoupling limit $g_s\, l_s^3 =$ fixed then the ten-dimensional Planck length $l_{10} = g_s\, l_s^{1/4}$ vanishes which if the ten-dimensional description would have been valid at all energy scales would imply a decoupling of gravity. However, the value of the eleven-dimensional Planck length $l_{11} = g_s^{1/3} l_s$ is fixed in this limit and the eleven-dimensional supergravity does not decouple. In contrast, when the B-field is non vanishing there are two cases where we can trust the ten-dimensional description at all energy scales [6]. It happens when the rank of the B-field is four or six.

There is however a second puzzle, which cannot be resolved in this way. Looking at the expansions at $r \to 0$ and $r \to \infty$ of $h_2(r)$ we see that it has wave-like solution that propagates only in one direction. The conservation of flux therefore requires that at every point in r the flux associated with $h_2(r)$ should be the same. This is clearly not the case as we see from (3.30). In fact the flux is everywhere the same but jumps to another value at $r = 0$. This means that there is an absorption (or emission) by the D6-branes and in fact in the so called decoupling limit it is actually a much stronger

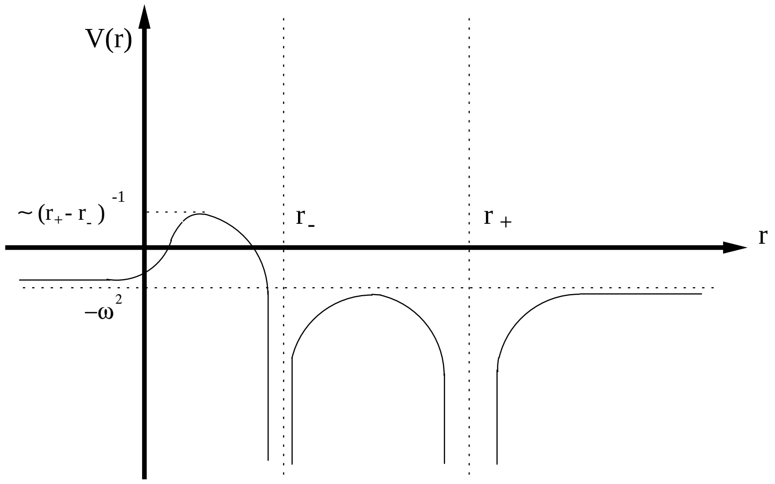

Figure 2: The graviton scattering potential in the non-extremal D6-branes background.

absorption as no flux exists at $r = 0$. Therefore the graviton absorption computation indicates that gravity does not decouple from the D6-branes worldvolume theory in the presence of a non-vanishing B-field.

We should note however that since our computation has been done in the framework of supergravity there is still the issue of the resolution of the singularity at $r = 0$. One way to attempt at resolving this singularity is to use the non-extremal D6-branes solution [19]

$$\begin{aligned} ds^2 &= -\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)^{-1/2} dt^2 + \\ &\quad + \left(1-\frac{r_+}{r}\right)^{-1}\left(1-\frac{r_-}{r}\right)^{1/2} dr^2 \left(1-\frac{r_-}{r}\right)^{1/2} dx_i^2 + r^2\left(1-\frac{r_-}{r}\right)^{3/2} d\Omega_2^2\,, \\ e^{-2\phi} &= \left(1-\frac{r_-}{r}\right)^{-3/2}\,. \end{aligned} \tag{3.31}$$

As before, we decompose the graviton $h\,(r) = g(r)\,c\,(r)$ with

$$c\,(r) = \frac{1}{\left(r(r-r_+)\right)^{1/2}}\,. \tag{3.32}$$

Then the equation for $g(r)$ takes the form (3.10) with the potential

$$V(r) = -\omega^2\,\frac{r\,(r-r_-)}{(r-r_+)^2} - \frac{(r_- - r_+)^2}{(r-r_-)^2\,(r-r_+)^2} \tag{3.33}$$

depicted in figure 2.

The physical region is $r \geq r_+$. The solutions of near $r = r_+$ are $g(r) \sim (r-r_+)^{\pm i\beta}$, where $\beta = \sqrt{1+\omega^2 r_+(r_+-r_-)}$. At infinity the solutions take the form $g(r) \sim e^{\pm i\omega r}$. Both solutions are normalizable, and in the extremal limit $r_+ = r_-$ we do not approach the non-normalizable solution. Thus, the proper way of resolving the singularity requires a better understanding of the inclusion of the D6-branes degrees of freedom. Details of such resolution may depend on the B-field but it seems unlikely that they will modify our conclusion about the non decoupling of gravity. For comparison we will analyse in the next section other cases where decoupling exists but there is a singularity at $r = 0$.

4. Graviton scattering on Dp-branes

In this section we study the scattering of gravitons on Dp-branes ($p < 5$) in the supergravity description. We analyse the scattering potentials, the absorption cross section and discuss the issue of normalizability of scattering waves. The purpose of this discussion is to compare with the D6-branes other cases where the bulk gravity does decouple from the brane modes while the supergravity solution is singular.

4.1 Dp-branes

The Ricci scalar $\mathcal{R}_p$ and the dilaton Φ of the Dp-branes supergravity background are

$$\mathcal{R}_p \sim (p-3)\,\frac{R_p^{2\,(7-p)}}{r^{2\,(8-p)}\, f_p^{5/2}(r)}\,, \qquad e^{\Phi} \sim f_p^{\frac{3-p}{4}}(r)\,, \tag{4.1}$$

where $f_p = 1 + \frac{R_p^{7-p}}{(7-p)r^{7-p}}$ is the harmonic function and $R_p^{7-p} \sim g_s\, N\, l_s^{7-p}$. When $p > 3$ the Ricci scalar diverges at $r = 0$, while when $p < 3$ the dilaton blows up.

As before, the transversal graviton modes $h(r)$ are related to the minimally couple scalar $\varphi(r)$ like

$$h(r) = f_p^{-1/2}(r)\,\varphi(r)\,. \tag{4.2}$$

Consider the minimally coupled scalars. Scattering waves

$$\varphi(t, r, \Omega), = \phi_{l,m_1,\ldots,m_{7-p}}(r)\, e^{i\omega t}\, Y_{l,m_1,\ldots,m_{7-p}}(\Omega) \tag{4.3}$$

can be calculated by solving the differential equation

$$\left[r^{-8+p}\,\partial_r\, r^{8-p}\,\partial_r + \omega^2\, f_p(r) - \frac{l(l+(7-p))}{r^2}\right]\phi_{l,m_1,\ldots,m_{7-p}}(r) = 0\,, \tag{4.4}$$

for the radial factor $\phi(r)_{l,m_1,\ldots,m_{7-p}}$. Let $\phi_{l,m_1,\ldots,m_{7-p}}(\rho/\omega) = \psi(\rho)/(\omega^2\rho^{(8-p)/2)})$ and $\rho = \omega r$ one finds the Schrödinger-like equation

$$\partial_\rho^2\,\psi(\rho) - V_p(\rho)\,\psi(\rho) = 0\,, \tag{4.5}$$

where

$$V_p(\rho) = -\left(1 + \frac{(\omega R_p)^{7-p}}{(7-p)\rho^{7-p}}\right) + \frac{(p-8)(p-6)}{4\,\rho^2} + \frac{l(l+7-p)}{\rho^2}\,. \tag{4.6}$$

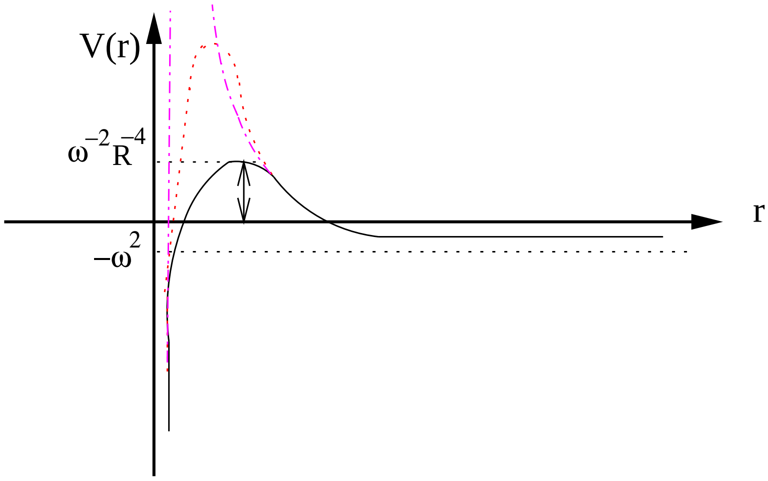

Figure 3: The graviton scattering potential in Dp-branes background. The value of the height $\omega^{-2}R^{-4}$ corresponds to $p=3$.

Let us discuss the potential $V_p(\rho)$, depicted in figure 3. Consider for simplicity the s-wave. As ρ approaches zero the potential diverges to minus infinity $V(\rho \to \infty) \sim -1/\rho^{7-p}$. As ρ approaches infinity it converges to -1: $V(\rho \to \infty) = -1$. In between it reaches a maximum at

$$\rho_{\max} = \left(\frac{2(7-p)(\omega R_p)^{7-p}}{(p-8)(p-6)} \right)^{\frac{1}{5-p}} . \tag{4.7}$$

In the decoupling limit ωR_p goes to zero and the maximum value of the potential $V(\rho_{\max})$ goes to infinity

$$V\left(\rho_{\max}\right) \sim \frac{1}{(\omega R_p)^{\frac{2(7-p)}{5-p}}} , \quad \omega R_p \longrightarrow 0 , \tag{4.8}$$

creating an infinitely high barrier. In fact from $r=\infty$ to the radius $r_{\max}$ the potential looks like the one of a free wave in flat space time, written in polar coordinates with $8-p$ angles. In the limit the potential barrier makes it impossible for scattering waves to reach $r=0$. The bulk modes decouple because of the potential around $r=r_{\max}$. In comparison to the D6-branes we see that there the barrier part is missing in the potential.

The absorption cross section per unit world volume of Dp-branes ($p<5$) goes to zero in the decoupling limit. It is calculated in terms of fluxes of scattering solutions of (4.5).

$$\sigma_p(\omega, R_p) = \frac{(2\pi)^{8-p}}{\omega^{8-p}\,\Omega_{8-p}} \frac{\mathcal{F}_0^{\rm in}}{\mathcal{F}_\infty^{\rm in}} , \qquad \Omega_d = \frac{2\pi^{(d+1)/2}}{\Gamma\left(\frac{d+1}{2}\right)} , \tag{4.9}$$

$$\mathcal{F}_S = \int_S J^a \, d\sigma_a , \tag{4.10}$$

JHEP06(2000)002

p	$\rho \ll 1$	z	$\rho \gg (R_p\omega)^{5-p}$	$\sigma_p(\omega, R_p)$
1	$i\,(z)^{3/2} H^1_{3/2}(z)$	$(\omega R_1)^3/\rho^2$	$48\sqrt{2/\pi}\,\rho^{-3} J_3(\rho)$	$\frac{2\,\pi^4}{3}\,\omega^2 R_1^9$
2	$i\,(z)^{5/3} H^1_{5/3}(z)$	$(2/3)(\omega R_2)^{5/2}/\rho^{3/2}$	$c_0\,\rho^{-5/2} J_{5/2}(\rho)$	$c_1 \omega^{2+1/3} R_2^{8+1/3}$
3	$i\,(z)^2 H^1_2(z)$	$(\omega R_3)^2/\rho$	$\frac{32}{\pi}\,\rho^{-2} J_2(\rho)$	$\frac{\pi^4}{8}\,\omega^3 R_3^8$
4	$i\,(z)^3 H^1_3(z)$	$2(\omega R_4)^{3/2}/\sqrt{\rho}$	$24\sqrt{2/\pi}\,\rho^{-3/2} J_{3/2}(\rho)$	$\frac{2\pi^3}{3}\,\omega^5 R_4^9$

Table 1: Dp-branes absorption cross sections.

$$\mathcal{F}_{S_{r=r_S}} = \int_{S_{r=r_S}} \sqrt{g}\, e^{-2\Phi}\, g^{rr}\, i\left(\partial_r\, \varphi^*\, \varphi - \varphi^*\, \partial_r\, \varphi\right) dx_{\|}^{p+1}\, d\Omega_{8-p}\,. \tag{4.11}$$

Here $S_{r=r_S}$ is the surface of all events of space time with $r = r_S$, $d\,\Omega_{8-p}$ is the volume element of the $(8-p)$-sphere. It is difficult to write a closed solution to eq. (4.5). One can solve for $\rho \ll 1$ and for $\rho \gg (R_p\,\omega)^{\alpha_p}$ ($\alpha_p > 1$). Assuming that $(R_p\,\omega)^{p-5} \ll 1$, the two regions overlap and the asymptotic solutions can be matched.[1] We summarize the results of the asymptotic solutions to the scattering equation in the background of Dp-branes and the absorption cross section in table 1.

$H^1_\nu(z) = J_\nu(z) + iN_\nu(z)$, $J_\nu(z)$ and $N_\nu(z)$ denote the Hankel, Bessel and Neumann functions, respectively. (The constants c_0, c_1 can be calculated to $c_0 = \sqrt{\pi/2}\frac{5\;\,2^{11/3}}{\Gamma(1/3)\sqrt{3}}$, $c_1 = \frac{\pi^3\,\Gamma(1/3)^2}{\sqrt[3]{3}\;2^2\;5}$.) The linear combinations of the wave like solutions are uniquely determined by the physical boundary condition $F^{\rm out}_{r=0} = 0$. The absorption cross section in string units vanishes in the decoupling limit

$$\sigma_p(\omega, R_p)/l_s^{8-p} \longrightarrow 0\,. \tag{4.12}$$

The scattering solutions found above are not normalizable in the norm induced from flux conservation

$$\mathcal{F}\left(S_{t=t_S}\right) = \int_{S_{r=r_S}} \sqrt{g}\, e^{-2\phi}\, g^{tt}\, i\Big(\partial_t \varphi^* \varphi - \varphi^* \partial_t \varphi\Big) dx_{\|}^{p}\, d\hat{x}_0\, dr\, d\Omega_{8-p}\,. \tag{4.13}$$

Their current density blows up at $r = 0$. Using the identities

$$\phi(r) \overset{r\to 0}{\sim} z^\nu\, H^1_\nu(z)\,, \tag{4.14}$$

$$H^1_\nu(z \to \infty) \;=\; \sqrt{\frac{2}{\pi z}}\, e^{i(z-\pi\nu/2-\pi/4)} \left(1 + O\left(\frac{1}{z}\right)\right), \tag{4.15}$$

$$\nu \;=\; \frac{7-p}{5-p}\,, \qquad z = \frac{5-p}{2}\,\frac{(\omega R_p)^{(7-p)/2}}{\rho^{(5-p)/2}}\,, \tag{4.16}$$

$$\sqrt{g}\, e^{-2\Phi}\, g^{tt}(r) \;=\; f_p(r)\, r^{8-p}\, \Omega_{8-p} \xrightarrow{r\to 0} r\, \frac{R_p^{7-p}}{(7-p)}\, \Omega_{8-p}\,, \tag{4.17}$$

[1]Explicit calculations of these types can be found in [22].

the flux density of $\mathcal{F}_t(r \to 0)$ diverges as

$$\sqrt{g}\, e^{-2\Phi}\, g^{tt}\, i\left(\partial_t\, \varphi^* \varphi - \varphi^* \partial_t\, \varphi\right) \overset{r \to 0}{\sim} r^{(5-p)/2}\, r\, r^{p-7} = \frac{1}{r^{(7-p)/2}}\,. \tag{4.18}$$

So it is not integrable for $p < 5$. This is related to the fact that generalized eigenfunctions are at most δ-normalizable.

Let us sketch why a gaussian wave packed at zero behaves nicely.

$$\begin{aligned} \phi(r) &\sim r^{\alpha} \int d\omega\, \frac{1}{\pi}\, e^{-(\omega-\omega_0)^2}\, e^{i\omega\left(t + R_p^{(7-p)/2}/r^{5-p}\right)} \\ &\sim r^{\tilde{\alpha}}\, e^{i\omega_0\left(t + R_p^{(7-p)/2}/r^{p-5}\right)}\, e^{-\left(t + R_p^{(7-p)/2}/r^{p-5}\right)^2}\,. \end{aligned} \tag{4.19}$$

In this way at $r \to 0$ superposition produces a damping factor due to the negative power $R_p^{(7-p)/2}/r^{p-5}$ in the exponent. Thus, although the physical boundary conditions forced to construct non-normalizable wave functions, appropriate superpositions resolve divergences. In fact this behaviour is due to the horizons being null. Note in comparison that D6 wave solutions cannot be superposed to give a damping factor as above — it has a timelike horizon.

To summarize, although for the Dp-branes with $p \neq 3$ the background is singular at $r = 0$, when $p < 5$ they differ from the D6-branes in several aspects. The scattering potential for the gravitons in the Dp-branes supergravity background has a barrier that goes to infinity in the decoupling limit. The scattering potential in the D6-branes cases does not have such behaviour. In both cases the non-normalizable solution leads to absorption. However, the absorption goes to zero in the decoupling limit for the Dp-branes but is non zero in the D6-branes cases. Furthermore the non-normalizable solution is δ-normalizable for Dp-branes but not for D6-branes.

The computation done in this section can be repeated with a rank $2m$ B-field. As in the D6-branes cases, the scattering potential is the same with $R_p^{7-p} \sim g_s\, l_s N \prod_{i \text{ odd}}^{2m-1} \cos^{-1} \alpha_i$ [6].

4.2 NS5-branes

The case of type-II NS5-branes or D5-branes is different from both Dp-branes ($p < 5$) and D6-branes. In some sense this case is the border between these two cases. The scattering potential of the Schrödinger-like equation is

$$V(r) = -\omega^2 + \left(\frac{3}{4} - \omega^2 R^2\right) \frac{1}{r^2}\,, \tag{4.20}$$

where $R = N l_s^2$. The sign of the second terms of the potential changes at $\omega = \frac{\sqrt{3}}{2R}$ and therefore the shape of potential changes too as in figure 4. The gravitons with energy less than $\omega < \frac{\sqrt{3}}{2R}$ see a barrier potential and cannot reach the branes.

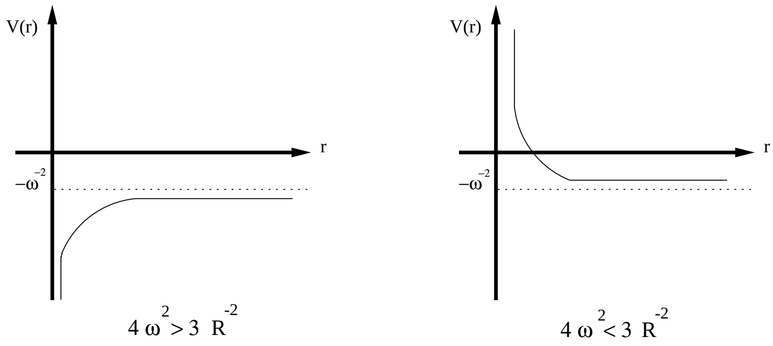

Figure 4: The scattering potential for gravitons in NS5-branes background.

Setting $\rho = \omega r$, the graviton scattering differential equation reads

$$\frac{\partial^2 \psi}{\partial \rho^2} + \frac{3}{\rho}\frac{\partial \psi}{\partial \rho} + \frac{R^2 \omega^2}{\rho^2}\psi = 0\,. \tag{4.21}$$

The equation has solutions ρ^α, where

$$\alpha = -1 \pm \sqrt{1-(R\omega)^2}\,. \tag{4.22}$$

For $R\,\omega > 1$, where α becomes imaginary, there is a wave like solution and. There is a non-zero absorption cross section even in the decoupling limit [23] for the particles with energy $\omega > m_s/\sqrt{N}$

The analysis of the graviton scattering for the NS5-branes in the presence of an RR field is the same. As before, the only change is $R = N l_s^2 \prod_i \cos^{-1} \alpha_i$ [24, 6].

Acknowledgments

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A. Type-IIA supergravity equations

The bosonic part of the low-energy effective action of the type-IIA string theory is

$$\begin{aligned} S_{\rm IIA} = \frac{1}{2\kappa^2}\int dx^{10}\sqrt{-G}\,\Bigg(e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}\,H^2\right] - \frac{1}{2\cdot 2!}\,(F_2)^2 - \\ - \frac{1}{2\cdot 4!}\left(\tilde{F}_4\right)^2\Bigg) - \frac{1}{4\kappa^2}\int B\,F_4\,F_4\,, \end{aligned} \tag{A.1}$$

where

$$F_n = dA_n\,, \qquad H = dB\,, \qquad \tilde{F}_4 = F_4 - H\wedge A_1\,. \tag{A.2}$$

B is the NS 2-form and A_n denote the the RR n-form. Varying the above action with respect to all the potentials we have:

$$D^k F_{ki} = -\frac{1}{3!}\, F_{klmi}\, H^{klm} - \frac{1}{3!}\, H_{klm}\, H^{klm}\, A_i + \frac{1}{2!}\, H_{ilm}\, H^{lmn}\, A_n\,, \tag{A.3}$$

$$D^k\left[F_{kijl} - 4H_{[ijk}\, A_{l]}\right] = \frac{1}{3!\cdot 4!}\, \varepsilon_{ijlmnopqrs}\, H^{mno}\, F^{pqrs}\,, \tag{A.4}$$

$$\begin{aligned} &D^k\left[e^{-2\phi}\, H_{kij} + H_{kij}\, A_l\, A^l - 3A_{[i}\, H_{jk]l}\, A^l - F_{kijl}\, A^l\right] = \\ &\quad = \frac{1}{2\cdot 4!\cdot 4!}\, \varepsilon_{kijlmnopqrs}\, F^{lmno}\, F^{pqrs}\,, \end{aligned} \tag{A.5}$$

$$D_l\, \partial^l \phi = -\frac{1}{4}\, R + \frac{1}{4\cdot 2\cdot 3!}\, H_{ijk}\, H^{ijk} + \partial_k\, \phi\, \partial^k \phi\,, \tag{A.6}$$

$$\begin{aligned} R_{ij} = &-2D_i\, \partial_j \phi + \frac{3}{2\cdot 3!}\, H_{(i}{}^{lm}\, H_{j)lm} + \\ &+ e^{2\phi}\, \frac{1}{2\cdot 2!}\left(2(F_2)_{ik}\, (F_2)_j{}^k - \frac{1}{2}\, g_{ij}\, (F_2)_{kl}(F_2)^{kl}\right) + \\ &+ e^{2\phi}\, \frac{1}{2\cdot 4!}\left(4\, (\tilde{F}_4)_{(i}{}^{lmn}(\tilde{F}_4)_{j)lmn} - \frac{1}{2}\, g_{ij}(\tilde{F}_4)_{klmn}(\tilde{F}_4)_{klmn}\right). \end{aligned} \tag{A.7}$$

The linearized field equations of the dilaton and the Ricci tensor take the form

$$\begin{aligned} D_l\, \partial^l \phi - \Gamma(h)_l\, \partial^l \dot{\phi} - h^{lm}\, D_l\, \partial_m\, \dot{\phi} = &\frac{1}{4}\, R_{ij}\, h^{ij} - \frac{1}{4}\left(D_j\, D_i\, h^{ij} - D^2\, h_j^j\right) + \\ &+ \frac{2}{4\cdot 2\cdot 3!}\, H_{ijk}\, \dot{H}^{ijk} - \frac{3}{4\cdot 2\cdot 3!}\, h^{il}\, \dot{H}_{ijk}\, \dot{H}_l^{jk} + \\ &+ 2\, \partial_k\, \dot{\phi}\, \partial^k \phi - h^{kl}\, \partial_k\, \dot{\phi}\, \partial_l \dot{\phi}\,, \end{aligned} \tag{A.8}$$

$$\begin{aligned} &D_{(i} D_a\, h^a{}_{j)} - \frac{1}{2}\, D^2 h_{ji} - \frac{1}{2}\, D_j D_i\, h_a^a + (R_g)_{m(i}\, h_{j)}^m + (R_g)_{jmni}\, h^{mn} = \\ &\quad = -2D_i\, \partial_j\, \phi + 2\, \Gamma(h)_i\, \partial_j\, \dot{\phi} + \frac{2\cdot 3}{2\cdot 3!}\, \dot{H}_{(i}{}^{lm} H_{j)lm} - \frac{2\cdot 3}{2\cdot 3!} h^{kl}\, \dot{H}_{(i|k}{}^{m} \dot{H}_{j)lm} + \\ &\quad + 2\, \phi\, e^{2\dot{\phi}}\Bigg(\frac{1}{2\cdot 2!}\left[2\left(\dot{F}_2\right)_{(i}{}^{l}\left(\dot{F}_2\right)_{j)l} - \frac{1}{2}\, g_{ij}\left(\dot{F}_2\right)^{lm}\left(\dot{F}_2\right)_{lm}\right] + \\ &\qquad + \frac{1}{2\cdot 4!}\left[4\left(\dot{\tilde{F}}_4\right)_{(i}{}^{lmn}\left(\dot{\tilde{F}}_4\right)_{j)lmn} - \frac{1}{2}\, g_{ij}\left(\dot{\tilde{F}}_4\right)^{lmno}\left(\dot{\tilde{F}}_4\right)_{lmno}\right]\Bigg) + \\ &\quad + e^{2\dot{\phi}}\Bigg(\frac{1}{2\cdot 2!}\left[2\cdot 2\left(\dot{F}_2\right)_{(i}{}^{l}\, (F_2)_{j)l} - \frac{2}{2}\, g_{ij}\left(\dot{F}_2\right)^{lm}(F_2)_{lm}\right] + \\ &\qquad + \frac{1}{2\cdot 4!}\Bigg[4\cdot 2\left(\dot{\tilde{F}}_4\right)_{(i}{}^{lmn}\left(F_4 - \dot{H} A_1 - H \dot{A}_1\right)_{j)lmn} - \\ &\qquad\quad - \frac{2}{2}\, g_{ij}\left(\dot{\tilde{F}}_4\right)^{lmno}\left(F_4 - \dot{H} A_1 - H \dot{A}_1\right)_{lmno}\Bigg]\Bigg) - \end{aligned}$$

$$
\begin{aligned}
&- e^{2\dot\phi}\Bigg(\frac{1}{2\cdot 4!}\left[3\cdot 4h^{kl}\left(\dot{\tilde F}_4\right)_{(i|k}{}^{mn}\left(\dot{\tilde F}_4\right)_{j)lmn} - \frac{4}{2}\,g_{ij}\,h^{kl}\,\dot{\tilde F}_{kmno}\,\dot{\tilde F}_l{}^{mno}\right] + \\
&\qquad + \frac{1}{2\cdot 2!}\left[2\,h^{kl}\left(\dot F_2\right)_{(i|k}\left(\dot F_2\right)_{j)l} - \frac{2}{2}\,g_{ij}\,h^{kl}\left(\dot F_2\right)_k{}^m\left(\dot F_2\right)_{lm} + \right. \\
&\qquad\qquad \left. + \frac{1}{2}\,h_{ij}\left(\dot{\tilde F}_4\right)^{lmno}\left(\dot{\tilde F}_4\right)_{lmno} + \frac{1}{2}\,h_{ij}\left(\dot F_2\right)^{lm}\left(\dot F_2\right)_{lm}\right]\Bigg)\,,
\end{aligned}
\tag{A.9}
$$

where the dotted fields represent the background fields and $\Gamma(h)$ stands for contractions of the h_{ij} dependent part in the Christoffel symbols

$$
\left(\Gamma(h)\right)^i{}_{jk} = \frac{1}{2}\,g^{il}\Big(D_j\,h_{lk} + D_k\,h_{lj} - D_l\,h_{jk}\Big) + O\left(h^2\right). \tag{A.10}
$$

B. Type-IIB supergravity equations

The bosonic part of the low-energy effective action of the type-IIB string theory is

$$
\begin{aligned}
S_{\mathrm{IIB}} = \frac{1}{2\kappa^2}\int dx^{10}\sqrt{-G}\,\Bigg(&e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}\,H^2\right] - \frac{1}{2\cdot 3!}\left(\tilde F_3\right)^2 - \\
&- \frac{1}{2}\left(F_1\right)^2 - \frac{1}{4\cdot 5!}\left(\tilde F_5\right)^2\Bigg) - \frac{1}{4\kappa^2}\int A_4\,F_3\,H\,,
\end{aligned}
\tag{B.1}
$$

we impose the self duality condition $F_5 = *F_5$ at the level of the field equations, and

$$
\begin{aligned}
&F_n = dA_n\,, \qquad H = dB\,, \qquad \tilde F_3 = F_3 - H A_0\,, \\
&\tilde F_5 = F_5 - \frac{1}{2}\Big(B\wedge F_3 - A_2\wedge H\Big)\,.
\end{aligned}
\tag{B.2}
$$

Varying the above action with respect to all the potentials we have:

$$
\nabla_l\,\partial^l A_0 = -\frac{1}{3!}\,H_{ijk}\left(F_3 - A_0 H\right)^{ijk}, \tag{B.3}
$$

$$
\nabla^k\left(F_3 - A_0 H\right)_{kij} = \frac{1}{3!}\left(F_5\right)_{ijklm}H^{klm}, \tag{B.4}
$$

$$
\nabla_k\Big[\left(e^{-2\phi} + (A_0)^2\right)H - A_0 F_3\Big]_{kij} = -\frac{1}{3!}\left(F_5\right)_{ijklm}\left(F_3\right)^{klm}, \tag{B.5}
$$

$$
\nabla_l\,\partial^l\phi = -\frac{1}{4}\,R + \frac{1}{4\cdot 2\cdot 3!}\,H_{ijk}\,H^{ijk} + \partial_k\,\phi\,\partial^k\phi\,, \tag{B.6}
$$

$$
\begin{aligned}
R_{ij} = -2D_i\,\partial_j\,\phi &+ \frac{3}{2\cdot 3!}\,H_{(i}{}^{lm}H_{j)lm} + e^{2\phi}\frac{1}{2}\left(\left(F_1\right)_i\left(F_1\right)_j - \frac{1}{2}\,g_{ij}\left(F_1\right)^k\left(F_1\right)_k\right) + \\
&+ e^{2\phi}\frac{1}{4\cdot 4!}\left(\tilde F_5\right)_{(i}{}^{lmnp}\left(\tilde F_5\right)_{j)lmnp} + \\
&+ e^{2\phi}\frac{1}{2\cdot 3!}\left[3\left(\tilde F_3\right)_{(i}{}^{lm}\left(\tilde F_3\right)_{j)lm} - \frac{1}{2}\,g_{ij}\left(\tilde F_3\right)^{jlm}\left(\tilde F_3\right)_{jlm}\right],
\end{aligned}
\tag{B.7}
$$

$$\left(\tilde{F}_5\right)_{ijklm} = \frac{1}{5!}\,\varepsilon_{ijklmnopqr}\left(\tilde{F}_5\right)^{opqrs}\,, \tag{B.8}$$

$$6\,\partial_{[i}\left(\tilde{F}_5\right)_{jklmn]} = \frac{6!}{3!\cdot 3!}\,(F_3)_{[ijk}\,(H_3)_{lmn]}\,. \tag{B.9}$$

The linearized field equations of the dilaton and the Ricci tensor take the form

$$\begin{aligned} D_l\,\partial^l\phi - \Gamma(h)_l\,\partial^l\dot{\phi} - h^{lm}D_l\,\partial_m\,\dot{\phi} &= \frac{1}{4}\,R_{ij}\,h^{ij} - \frac{1}{4}\Big(D_j\,D_i\,h^{ij} - D^2\,h^j_j\Big) + \\ &\quad + \frac{2}{4\cdot 2\cdot 3!}\,H_{ijk}\,\dot{H}^{ijk} - \frac{3}{4\cdot 2\cdot 3!}\,h^{il}\,\dot{H}_{ijk}\,\dot{H}_l{}^{jk} + \\ &\quad + 2\,\partial_k\,\dot{\phi}\,\partial^k\phi - h^{kl}\,\partial_k\,\dot{\phi}\,\partial_l\,\dot{\phi}\,, \end{aligned} \tag{B.10}$$

$$\begin{aligned} &D_{(i}\,D_a\,h^a{}_{j)} - \frac{1}{2}\,D^2\,h_{ji} - \frac{1}{2}\,D_j\,D_i\,h^a_a + (R_g)_{m(i}\,h^m_{j)} + (R_g)_{jmni}\,h^{mn} = \\ &= -2\,D_i\,\partial_j\phi + 2\,(\Gamma(h))_i\,\partial_j\,\dot{\phi} + \frac{2\cdot 3}{2\cdot 3!}\,\dot{H}_{(i}{}^{lm}\,H_{j)lm} - \frac{2\cdot 3}{2\cdot 3!}\,h^{kl}\,\dot{H}_{(i|k}{}^{m}\,\dot{H}_{j)lm} + \\ &\quad + \,2\,\phi\,e^{2\dot{\phi}}\left(\frac{1}{4\cdot 4!}\left(\dot{\tilde{F}}_5\right)_{(i}{}^{lmnp}\left(\dot{\tilde{F}}_5\right)_{j)lmnp} + \right. \\ &\qquad + \frac{1}{2\cdot 3!}\left[3\left(\dot{\tilde{F}}_3\right)_{(i}{}^{lm}\left(\dot{\tilde{F}}_3\right)_{j)lm} - \frac{1}{2}\,g_{ij}\left(\dot{\tilde{F}}_3\right)^{klm}\left(\dot{\tilde{F}}_3\right)_{klm}\right] + \\ &\qquad \left. + \frac{1}{2}\left[\left(\dot{F}_1\right)_{(i}\left(\dot{F}_1\right)_{j)} - \frac{1}{2}\,g_{ij}\left(\dot{F}_1\right)^k\left(\dot{F}_1\right)_k\right]\right) + \\ &\quad + \,e^{2\dot{\phi}}\left(\frac{2}{4\cdot 4!}\left(\dot{\tilde{F}}_5\right)_{(i}{}^{lmnp}\left(F_5 - \frac{1}{2}\Big(B\,\dot{F}_3 + \dot{B}\,F_3 - A_2\,\dot{F}_3 - \dot{A}_2\,F_3\Big)\right)_{j)lmnp} + \right. \\ &\qquad + \frac{1}{2\cdot 3!}\left[3\cdot 2\left(\dot{\tilde{F}}_3\right)_{(i}{}^{lm}\left(F_3 - A_0\dot{H} - \dot{A}_0 H\right)_{j)lm} - \right. \\ &\qquad\qquad \left. - \frac{2}{2}\,g_{ij}\left(\dot{\tilde{F}}_3\right)^{klm}\left(F_3 - A_0\,\dot{H} - \dot{A}_0\,H\right)_{klm}\right] + \\ &\qquad \left. + \frac{1}{2}\left[2\left(\dot{F}_1\right)_{(i}(F_1)_{j)} - \frac{2}{2}\,g_{ij}\left(\dot{F}_1\right)^k(F_1)_k\right]\right) - \\ &\quad - \,e^{2\dot{\phi}}\left(\frac{4}{4\cdot 4!}\,h^{kl}\left(\dot{\tilde{F}}_5\right)_{(i|k}{}^{mnp}\left(\dot{\tilde{F}}_5\right)_{j)lmnp} + \right. \\ &\qquad + \frac{1}{2\cdot 3!}\left[2\cdot 3\,h^{kl}\left(\dot{\tilde{F}}_3\right)_{(i|k}{}^{m}\left(\dot{\tilde{F}}_3\right)_{j)lm} - \frac{3}{2}\,g_{ij}\,h^{kl}\left(\dot{\tilde{F}}_3\right)_k{}^{mn}\left(\dot{\tilde{F}}_3\right)_{lmn} + \right. \\ &\qquad\qquad \left. + \frac{1}{2}\,h_{ij}\left(\dot{\tilde{F}}_3\right)^{lmn}\left(\dot{\tilde{F}}_3\right)_{lmn}\right] + \\ &\qquad \left. + \frac{1}{2}\left[-\frac{1}{2}\,g_{ij}\,h^{kl}\left(\dot{F}_1\right)_k\left(\dot{F}_1\right)_l + \frac{1}{2}\,h_{ij}\left(\dot{F}_1\right)^l\left(\dot{F}_1\right)_l\right]\right). \end{aligned} \tag{B.11}$$

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